Principles of Programming Languages
Small examination

Student ID: Name:

Problem 1 Illustrate the quilts represented by the following expressions (1), (2), and (3) in the language Little Quilt.

(1)
```
sew (turn (turn (b)), a)
```

(2)
```
let
    val x = turn (b)
in
    sew (x,x)
end
```

(3)
```
let
    fun unturn (x) = turn (turn (turn (x)))
    fun pile (x,y) = unturn (sew (turn (y), turn (x)))
    val aa = pile (a, turn (turn (a)))
    val bb = pile (unturn (b), turn (b))
in
    sew (aa, bb)
end
```

The meaning of `a`, `b`, `turn`, `sew` are as follows. The other constructs of Little Quilt (`let` expressions, `val` declaration, `fun` declaration) have the meaning explained in the lecture.

- Expressions `a` and `b` represent the quilts in Figure 1 and Figure 2 respectively.

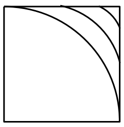

Figure 1: The quilt that `a` represents

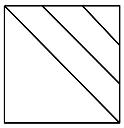

Figure 2: The quilt that `b` represents

- The expression `turn (`e`)` represents the quilt obtained by rotating 90 degrees to the right the quilt represented by the expression e.
- The expression `sew (`e_1`, `e_2`)` represents the quilt that is obtained by sewing the two quilts e_1 and e_2, where e_1 is in the left side and e_2 is in the right side, and they must have the same height.

Problem 2 Answer the following problems about the control flow in the imperative language presented in the lecture.

(1) Illustrate the control flow of the following program fragment.

```
if x>0 then x := x - 1
else if y>0 then y := y - 1
     else y := y + 1
```

(2) Illustrate the control flow of the following program fragment.

```
x := 10;
sum := 0;
L:  sum := sum + x;
x := x - 1;
if x>0 then
    goto L
```

(3) Illustrate the control flow of the following program fragment.

```
while x>0 do
    begin
        if x=3 then
            begin
                x := x - 1;
                continue
            end;
        y := y + 1;
        x := x - 1
    end
```

(4) Illustrate the control flow of the following program fragment.

```
while x>0 do
    begin
        while y>0 do
            begin
                if x=3 then
                    break;
                z := z + 1;
                y := y - 1
            end;
        x := x - 1
    end
```

(5) How many entries and exits does the if statement (`if x=3 then break;`) in the program fragment (4) have?

Problem 3 Derive the Hoare triples (1), (2), and (3) by using the rules presented in the lecture.

(1) $\{a = 3\}\ a := a + 1\ \{a = 4\}$

(2) $\{a = 3\}\ a := a + 1; a := a + 2\ \{a = 6\}$

(3) $\{a = 4\}$ **if** $a = 4$ **then** $a := a + 2$ **else** $a := a - 3\ \{a = 6\}$

(4) $\{a = 5\}$ **while** $a > 0$ **do** $a := a - 1\ \{a = 0\}$

Rules presented in the lecture

Hoare logic

$$\frac{\{P\}\ S_1\ \{Q\} \quad \{Q\}\ S_2\ \{R\}}{\{P\}\ S_1;\ S_2\ \{R\}}\ \text{(composition rule)}$$

$$\frac{\{P\ \wedge\ E\}\ S_1\ \{Q\} \quad \{P\ \wedge\ \neg\ E\}\ S_2\ \{Q\}}{\{P\}\ \textbf{if}\ E\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \{Q\}}\ \text{(conditional rule)}$$

$$\frac{\{P\ \wedge\ E\}\ S\ \{P\}}{\{P\}\ \textbf{while}\ E\ \textbf{do}\ S\ \{P\ \wedge\ \neg\ E\}}\ \text{(while rule)}$$

$$\frac{}{\{Q[E/x]\}\ x\ :=\ E\ \{Q\}}\ \text{(assignment axiom)}$$

$$\frac{P \Rightarrow P' \quad \{P'\}\ S\ \{Q'\} \quad Q' \Rightarrow Q}{\{P\}\ S\ \{Q\}}\ \text{(consequence rule)}$$