Princeples of programming languages 5: An answer to the exercise 4

Exercise 4 Derive the state after execurint the statement `while(Y){Y=(Y-20);}` in the state $\sigma = \{(\mathtt{X}, 10), (\mathtt{Y}, 40), (\mathtt{Z}, 30)\}$.

(An answer)

$$\dfrac{\dfrac{}{< \mathtt{Y}, \sigma > \to 40} \quad \dfrac{\dfrac{< \mathtt{Y}, \sigma > \to 40 \quad < \mathtt{20}, \sigma > \to 20}{< \mathtt{(Y-20)}, \sigma > \to 20}}{< \mathtt{Y=(Y-20);}, \sigma > \to \sigma[20/\mathtt{Y}]} \quad \dfrac{(*)}{< \mathtt{while(Y)\{Y=(Y-20);\}}, \sigma[20/\mathtt{Y}] > \to \sigma[20/\mathtt{Y}][0/\mathtt{Y}]}}{< \mathtt{while(Y)\{Y=(Y-20);\}}, \sigma > \to \sigma[20/\mathtt{Y}][0/\mathtt{Y}]}$$

We write the part $(*)$ with the relation below the line.

$$\dfrac{\dfrac{}{< \mathtt{Y}, \sigma[20/\mathtt{Y}] > \to 20} \quad \dfrac{\dfrac{< \mathtt{Y}, \sigma[20/\mathtt{Y}] > \to 20 \quad < \mathtt{20}, \sigma[20/\mathtt{Y}] > \to 20}{< \mathtt{(Y-20)}, \sigma[20/\mathtt{Y}] > \to 0}}{< \mathtt{Y=(Y-20);}, \sigma[20/\mathtt{Y}] > \to \sigma[20/\mathtt{Y}][0/\mathtt{Y}]} \quad \dfrac{< \mathtt{Y}, \sigma[20/\mathtt{Y}][0/\mathtt{Y}] > \to 0}{< \mathtt{while(Y)\{Y=(Y-20);\}}, \sigma[20/\mathtt{Y}][0/\mathtt{Y}] > \to \sigma[20/\mathtt{Y}][0/\mathtt{Y}]}}{< \mathtt{while(Y)\{Y=(Y-20);\}}, \sigma[20/\mathtt{Y}] > \to \sigma[20/\mathtt{Y}][0/\mathtt{Y}]}$$

By the above derivation, executing the statement `while(Y){Y=(Y-20);}` in the state σ terminates in the following state.

$$\sigma[20/\mathtt{Y}][0/\mathtt{Y}] = \{(\mathtt{X}, 10), (\mathtt{Y}, 0), (\mathtt{Z}, 30)\}$$