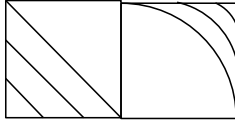


Principles of Programming Languages

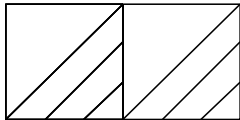
Answers for small examination 1

Problem 1 Illustrate the quilts represented by the following expressions (1), (2), and (3) in the language Little Quilt.

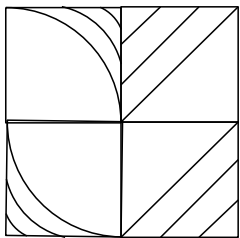
(1) `sew (turn (turn (b)), a)`



(2) `let`
 `val x = turn (b)`
`in`
 `sew (x,x)`
`end`



(3) `let`
 `fun unturn (x) = turn (turn (turn (x)))`
 `fun pile (x,y) = unturn (sew (turn (y), turn (x)))`
 `val aa = pile (a, turn (turn (a)))`
 `val bb = pile (unturn (b), turn (b))`
`in`
 `sew (aa, bb)`
`end`



The meaning of `a`, `b`, `turn`, `sew` are as follows. The other constructs of Little Quilt (`let` expressions, `val` declarations, `fun` declarations) have the meaning explained in the lecture.

- Expressions `a` and `b` represent the quilts in Figure 1 and Figure 2 respectively.
- The expression `turn (e)` represents the quilt obtained by rotating 90 degrees to the right the quilt represented by the expression `e`.
- The expression `sew (e1, e2)` represents the quilt that is obtained by sewing the two quilts `e1` and `e2`, where `e1` is in the left side and `e2` is in the right side, and they must have the same height.

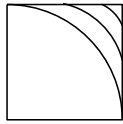


Figure 1: The quilt that **a** represents

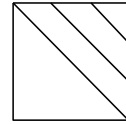


Figure 2: The quilt that **b** represents

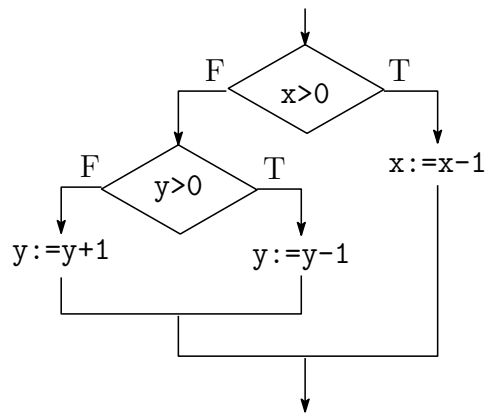
Problem 2 Answer the following problems about the control flow in the imperative language presented in the lecture.

- (1) Illustrate the control flow of the following program fragment.

```

if x>0 then x := x - 1
else if y>0 then y := y - 1
     else y := y + 1

```

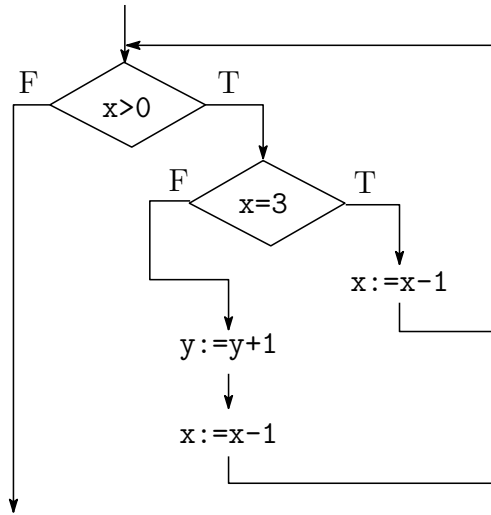


- (2) Illustrate the control flow of the following program fragment.

```

while x>0 do
  begin
    if x=3 then
      begin
        x := x - 1;
        continue
      end;
    y := y + 1;
    x := x - 1
  end
end

```



Problem 3

Derive the Hoare triples (1), (2), and (3) by using the rules presented in the lecture.

$$(1) \{a = 3\} a := a + 1 \{a = 4\}$$

$$\frac{a = 3 \Rightarrow a + 1 = 4 \quad \overline{\{a + 1 = 4\} a := a + 1 \{a = 4\}} \text{ (assign)} \quad a = 4 \Rightarrow a = 4 \text{ (conseq)}}{\{a = 3\} a := a + 1 \{a = 4\}}$$

As I said in the lecture, the logical expression $a = 4 \Rightarrow a = 4$ in the above proof tree may be omitted in this class as follows.

$$\frac{a = 3 \Rightarrow a + 1 = 4 \quad \overline{\{a + 1 = 4\} a := a + 1 \{a = 4\}} \text{ (assign)}}{\{a = 3\} a := a + 1 \{a = 4\}} \text{ (conseq)}$$

$$(2) \{a = 3\} a := a + 1; a := a + 2 \{a = 6\}$$

$$\frac{\frac{a = 3 \Rightarrow a + 1 = 4 \quad \overline{\{a + 1 = 4\} a := a + 1 \{a = 4\}} \text{ (assign)}}{\{a = 3\} a := a + 1 \{a = 4\}} \text{ (conseq)} \quad \frac{a = 4 \Rightarrow a + 2 = 6 \quad \overline{\{a + 2 = 6\} a := a + 2 \{a = 6\}} \text{ (assign)}}{\{a = 4\} a := a + 2 \{a = 6\}} \text{ (conseq)}}{\{a = 3\} a := a + 1; a := a + 2 \{a = 6\}} \text{ (composition)}$$

In this proof, I omitted $a = 4 \Rightarrow a = 4$ and $a = 6 \Rightarrow a = 6$ in the applications of the consequence rule.

$$(3) \{x = 5\} \text{ while } x > 0 \text{ do } x := x - 1 \{x = 0\}$$

Due to space restriction, I write the proof tree by separating it into two parts.

$$\frac{\text{(I write this part below.)} \quad \frac{x = 5 \Rightarrow x \geq 0 \quad \overline{\{x \geq 0\} \text{ while } x > 0 \text{ do } x := x - 1 \{x \geq 0 \wedge \neg x > 0\}} \quad x \geq 0 \wedge \neg x > 0 \Rightarrow x = 0 \text{ (conseq)}}{\{x = 5\} \text{ while } x > 0 \text{ do } x := x - 1 \{x = 0\}} \text{ (conseq)}}{\{x = 5\} \text{ while } x > 0 \text{ do } x := x - 1 \{x = 0\}}$$

$$\frac{\frac{x \geq 0 \wedge x > 0 \Rightarrow x - 1 \geq 0 \quad \overline{\{x - 1 \geq 0\} x := x - 1 \{x \geq 0\}} \text{ (assign)}}{\{x \geq 0 \wedge x > 0\} x := x - 1 \{x \geq 0\}} \text{ (conseq)} \quad x \geq 0 \Rightarrow x \geq 0 \text{ (conseq)}}{\overline{\{x \geq 0\} \text{ while } x > 0 \text{ do } x := x - 1 \{x \geq 0 \wedge \neg x > 0\}}} \text{ (while)}$$

In the above proof tree, the logical expression $x \geq 0 \Rightarrow x \geq 0$ may be omitted as follows.

$$\frac{\frac{x \geq 0 \wedge x > 0 \Rightarrow x - 1 \geq 0 \quad \overline{\{x - 1 \geq 0\} x := x - 1 \{x \geq 0\}} \text{ (assign)}}{\{x \geq 0 \wedge x > 0\} x := x - 1 \{x \geq 0\}} \text{ (conseq)}}{\{x \geq 0\} \textbf{while } x > 0 \textbf{ do } x := x - 1 \{x \geq 0 \wedge \neg x > 0\}} \text{ (while)}$$

I abbreviated the assignment axiom as assign, the consequence rule as conseq, the while rule as while, and the composition rule as composition.

Problem 4

Show the output produced by executing the following Pascal program. When the keyword `var` is attached to a formal parameter, it designates the parameter as call-by-reference. The procedure `writeln` writes out to the standard output the value of the parameter and a new line character.

```

program test;
var x : integer;
var y : integer;
procedure swap
  (var x: integer;
   var y : integer);
var z : integer;
begin
  z := x; x := y; y := z
end;
begin
  x := 3;
  y := 4;
  swap (x,y);
  writeln (x);
  writeln (y)
end.

```

4
3

Problem 5

Show the output produced by executing the following Pascal program. Note that Pascal is statically (lexically) scoped.

```

program P;
var n : char;
procedure W;
begin
  writeln(n)
end;
procedure D;
var n : char;
begin
  n := 'D';
  W
end;
begin
  n := 'L';
  D
end.

```

L
L

Problem 6

Show the meaning of the following programs (1) and (2) by using the rules presented in the lecture. Note that the programs are in the small subset of C presented in the lecture. Let the states before executing the programs both to be $\sigma = \{(X, 3), (Y, 1), (Z, 0)\}$.

(1) 2

$$\langle 2, \sigma \rangle \rightarrow 2$$

So in the state σ , executing the program 2 results in 2.

(2) $((2+3)*X)$

$$\frac{\frac{\langle 2, \sigma \rangle \rightarrow 2 \quad \langle 3, \sigma \rangle \rightarrow 3}{\langle (2+3), \sigma \rangle \rightarrow 5} \quad \langle X, \sigma \rangle \rightarrow 3}{\langle ((2+3)*X), \sigma \rangle \rightarrow 15}$$

So in the state σ , executing the program $((2+3)*X)$ results in 15.