(Course Code)	Applied Mathematics	Isao Sasano

College	College of Engineering
Department	Department of Information Science and Engineering
Grade	2nd Year Students
Semester	First Semester
Credit	2
Course Type	Elective
Course Classification	Specialty
Mode of Delivery	Lecture

Course Outline

Discrete Fourier transform (DFT) is used for processing sounds and graphics in digital computers. This lecture aims at being able to do Fourier series expansion, which forms the basis for DFT. As an introduction to Fourier series expansion we illustrate the least-square method and the orthogonal function expansion. Fourier series expansion is an instance of the orthogonal function expansion. Understanding Fourier series expansion forms the basis for understanding Fourier transform and DFT, which are topics covered in lectures of signal processing.

Achievement Objectives

1. Understanding the least-square method and being able to approximate given sequences of data or functions by linear functions or quadratic functions
2. Understanding orthogonal functions and being able to do the orthogonal function expansion for given functions by some given set of orthogonal functions
3. Understanding Gram-Schmidt orthogonalisation, which is a method (algorithm) for orthogonalise a set of vectors in an inner product space, and being able to construct an orthogonal set of functions from a given set of functions.
4. Being able to do Fourier series expansion, which is an important instance of the orthogonal function expansion.

Course Plan

【Course Plan】	【Assignment (including preparation and review) 】

1.	Introduction and the least-square method (1) ● Approximation of sequences of data in linear functions	Read Section 20.5 for the least-square method. As for methods for solving systems of linear equations, refer to Section 7.3 and 20.1 for the Gauss elimination (a direct method) and Section 20.3 for the Gauss-Seidel method (an iterative method).
2.	The least-square method (2) ● Approximation of sequences of data in quadratic functions	Refer to Example 2 in Section 20.5 for this example.
3.	The least-square method (3) ● Approximation of sequences of data in linear combination of some fixed set of functions	It is not treated in the reference book.
4.	The least-square method (4) ● Approximation of functions in linear combination of some fixed set of functions	Refer to Problem 14 in Section 20.5 for this topic.
5.	The least-square method (5) and the orthogonal function expansion (1) ● Approximation of column vectors ● Approximation of functions in linear combination of some fixed set of orthogonal functions ● An orthogonal set of functions --- Legendre polynomials	Read Section 11.5 for orthogonal functions. Refer to Section 5.2 for Legendre polynomials. Refer to Example 1 in Section 11.6 for function expansion by Legendre polynomials. Read Section 4.0 for column vectors.
6.	The orthogonal function expansion (2) ● An orthogonal set of functions --- Trigonometric functions ● The orthogonal function expansion	Read Section 11.6 for orthogonal function expansion, which is to obtain orthogonal series, or generalized Fourier series, of a given function. Refer to Section 11.5 for the definition of orthogonal functions. Refer to Appendix 3.1 for formulae about trigonometric functions (sine, cosine, and so on).
7.	The orthogonal function expansion (3) ● An example of the orthogonal function expansion --- Fourier series expansion	Read Section 11.1 for Fourier series expansion, which is to obtain Fourier series of a given

● Orthogonal set of functions with a weight function ● An example --- Chebyshev polynomials	function. Refer to Problem 14 in Section 11.5 for Chebyshev polynomials.
8. Mid-term examination and explanation of the answers ● Paper-and-pencil test for checking the understanding of the contents of the lectures from the first to the seventh	Review the contents of all the lectures until the last one.
9. The orghogonal function expansion (4) ● Examples --- Hermite polynomials and Laguerre polynomials	Refer to Problem 14 in Section 11.6 for Hermite polynomials. Refer to Problem 14 in Section 11.5 for Laguerre polynomials.
10. The orthogonal function expansion (5) ● The orthogonal function expansion in Chebyshev, Hermite, and Laguerre polynomials ● Inner product spaces ● An inner product space --- n-dimensional Euclidean space	Read Section 7.9 for the inner product spaces. Refer to Example 3 in Section 7.9 for the n-dimensional Euclidean space.
11. The orthogonal function expansion (6) ● Cauchy-Schwarz inequality ● Triangle inequality ● Orthonormal basis	Read Section 7.9 for Cauchy-Schwarz inequality and Triangle inequality. Read Section 7.4 for the definition of basis.
12. The orthogonal function expansion (7) ● Orthogonal projection ● Orthogonal basis ● Gram-Schmidt orthogonalisation	Read Section 9.2 for projections. Gram-Schmidt orthogonalisation is not treated in the reference book. Consult some linear algebra textbook.
13. The orthogonal function expansion (8) ● An example of Gram-Schmidt orthogonalisation	Gram-Schmidt orthogonalisation is not treated in the referece book. Consult some linear algebra textbook.
14. The orthogonal function expansion (9) ● Obtaining Legendre polynomials by Gram-Schmidt orthogonalisation	Gram-Schmidt orthogonalisation is not treated in the reference book. Consult some linear algebra textbook.
15. Final examination and explanation of the answers ● Paper-and-pencil test for checking the understanding of the contents of the lectures from the first to the fourteenth	Review the contents of all lectures.

Evaluation Method

and Criteria

Mid-term exam is evaluated on a 40-point scale, final exam a 50-point, and reports a 10-point. When the mid-term exam is M point, the final exam F point, and the repots R point, the overall score is R+M+F*(100-(R+M))/50.

Textbooks and Reference Materials

A reference book is:
Erwin Kreyszig, Advanced Engineering Mathematics, John Wiley & Sons Ltd; 10th edition, International Student Version, 2011.
Note that I do not use this book as a textbook. Note also that this book is thick and covers topics much more than this class covers.

Pre-Course Preparation

Basic knowledge of linear algebra and analysis

Office Hours, Contact Method

Before and after each lecture or any time agreed on by email

Relevance to Environmental Education

None