

A solution for Exercise 5

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Exercise Calculate $P_3(x)$ from the general form of Legendre polynomials.

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

Solution

$$\begin{aligned} P_3(x) &= \frac{1}{2^3 \cdot 3!} \frac{d^3}{dx^3} (x^2 - 1)^3 \\ &= \frac{1}{8 \cdot 6} \frac{d^3}{dx^3} (x^6 - 3x^4 + 3x^2 - 1) \\ &= \frac{1}{8 \cdot 6} \frac{d^2}{dx^2} (6x^5 - 12x^3 + 6x) \\ &= \frac{1}{8 \cdot 6} \frac{d}{dx} (30x^4 - 36x^2 + 6) \\ &= \frac{1}{8 \cdot 6} (120x^3 - 72x) \\ &= \frac{1}{8} (20x^3 - 12x) \\ &= \frac{1}{2} (5x^3 - 3x) \end{aligned}$$