Boolean-valued 2nd-order logic

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Joint work with Jouko Väänänen
Second order logic; background

Two semantics:

1. Full semantics: Highly complex (very powerful), does not enjoy completeness, \(\omega\)-compactness.

2. Henkin semantics: Very simple (very weak), enjoys completeness, \(\omega\)-compactness.
Second order logic; background

Two semantics:

1. Full semantics: Highly complex (very powerful), does not enjoy completeness, $\omega$-compactness.

2. Henkin semantics: Very simple (very week), enjoys completeness, $\omega$-compactness.

**Boolean valued second order logic** is a powerful logic sitting between the two semantics and might enjoy completeness.
2nd-order logic; Henkin models

\[
\begin{align*}
\text{Henkin models} & \quad \text{Models of ZFC} \\
\text{2nd-order logic} & \quad \text{Set theory}
\end{align*}
\]

**Definition**

A 2nd-order structure \( M = (X, G, \ldots) \) is a **Henkin model** if it satisfies Comprehension Axiom for each 2nd-order formula.
2nd-order logic; Henkin models

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\text{Henkin models} \quad \frac{\text{2nd-order logic}}{=} \quad \frac{\text{Models of ZFC}}{\text{Set theory}}
\]

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**Example**

A 2nd-order structure \( M = (X, \mathcal{P}(X), \ldots) \) is called a **full 2nd-order structure**.
2nd-order logic; Henkin models

\[ \frac{\text{Henkin models}}{2\text{-order logic}} = \frac{\text{Models of ZFC}}{\text{Set theory}} \]

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**Example**

A 2nd-order structure \( M = (X, \mathcal{P}(X), \ldots) \) is called a **full 2nd-order structure**.

**Theorem (Henkin)**

The semantics for 2nd-order logic given by Henkin models is sound and complete to a standard proof system in 2nd-order logic.
Corollary

The validity of 2nd-order logic via Henkin semantics is $\Sigma^0_1$.

Henkin semantics gives us a 2nd-order logic similar to 1st-order logic.
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Full semantics = semantics with full 2nd-order structures

Theorem (Väänänen)
The validity of 2nd-order logic via full semantics is $\Pi_2$-complete in ZFC.

Point: One can express the structures of the form $(V_\alpha, \in)$ via full 2nd-order structures.
Definition

Let $\mathcal{L}$ be a relational language. A **Boolean valued $\mathcal{L}$-structure** is a tuple $M = (A, \mathbb{B}, \{ R_i^M \})$ where

1. $A$ is a nonempty set,
2. $\mathbb{B}$ is a complete Boolean algebra, and
3. for each $n$-ary relational symbol $R_i$ in $\mathcal{L}$, $R_i^M : A^n \to \mathbb{B}$.
Boolean valued 2nd-order logic; Boolean valued structures

Definition

Let $\mathcal{L}$ be a relational language. A **Boolean valued $\mathcal{L}$-structure** is a tuple $M = (A, \mathbb{B}, \{ R^M_i \})$ where

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3. for each $n$-ary relational symbol $R_i$ in $\mathcal{L}$, $R^M_i : A^n \to \mathbb{B}$.

Example

If $\mathbb{B} = \{0, 1\}$, each $R^M_i$ is a relation in 1st-order logic and $M$ is the same as 1st-order structure.
Truth of 2nd-order formulas in Boolean valued structures

Basic idea: “subsets” are functions from $A$ to $\mathbb{B}$.

**Definition**

Let $M = (A, \mathbb{B}, \{R_i\})$ be a Boolean valued $\mathcal{L}$-structure. Then we assign $\|\phi[^{\vec{a}, \vec{f}}]\|_M^M \in \mathbb{B}$ to each 2nd-order formula $\phi$, $\vec{a} \in <\omega A$, and $\vec{f} \in <\omega (A\mathbb{B})$ as follows:

1. $\phi$ is $R_i(^{\vec{x}})$. Then $\|R_i(^{\vec{x}})[^{\vec{a}}]\|_M^M = R_i^M(^{\vec{a}})$.
2. $\phi$ is $X(x)$. Then $\|X(x)[^{a, \vec{f}}]\|_M^M = f(a)$.
3. Boolean combinations are as usual.
4. $\phi$ is $\exists x \psi$. Then $\|\exists x \psi[^{\vec{a}, \vec{f}}]\|_M^M = \bigvee_{b \in A} \|\psi[^{b, \vec{a}, \vec{f}}]\|_M^M$.
5. $\phi$ is $\exists X \psi$. Then $\|\exists X \psi[^{\vec{a}, \vec{f}}]\|_M^M = \bigvee_{g: A \rightarrow \mathbb{B}} \|\psi[^{\vec{a}, g, \vec{f}}]\|_M^M$. 
Boolean-valued 2nd-order logic; Boolean-valued structures ctd.

Boolean-valued structures = full 2nd-order structures in forcing extensions.
Boolean-valued 2nd-order logic; Boolean-valued structures ctd.

Boolean-valued structures = full 2nd-order structures in forcing extensions.

Remark

Given a Boolean-valued structure $\mathcal{M} = (A, \mathcal{B}, \{R^M_i\})$ and a $\mathcal{B}$-generic filter $G$ over $V$, the structure $\mathcal{M}$ corresponds to a full 2nd-order structure $\mathcal{M}[G] = (A, \mathcal{P}(X)^{V[G]}, \{R^M_i[G]\})$ in $V[G]$, where

$$R^M_i[G] = \{ x \in X, | R^M_i(x) \in G \}.$$
Boolean-valued 2nd-order logic; Boolean-valued structures ctd.

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Remark

Given a Boolean-valued structure $M = (A, \mathbb{B}, \{R_i^M\})$ and a $\mathbb{B}$-generic filter $G$ over $V$, the structure $M$ corresponds to a full 2nd-order structure $M[G] = (A, \mathcal{P}(X)^{V[G]}, \{R_i^{M[G]}\})$ in $V[G]$, where

$$R_i^{M[G]} = \{x \in X, | R_i^M(x) \in G\}.$$

For any 2nd-order sentence $\phi$, $||\phi||^M = 1$ iff $M[G] \models \phi$ for any $\mathbb{B}$-generic filter $G$ over $V$. 
Boolean valued 2nd-order logic; Boolean-validity

**Definition**

Let $\mathcal{L}$ be relational. A 2nd-order $\mathcal{L}$-sentence $\phi$ is **Boolean-valid** if $\|\phi\|^M = 1$ for any Boolean valued $\mathcal{L}$-structure $M$.

Our interest: $0^{2^b} = \{ \phi \mid \phi \text{ is Boolean-valid} \}$. 
Boolean valued 2nd-order logic; Boolean-validity

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Our interest: $0^{2b} = \{\phi \mid \phi \text{ is Boolean-valid}\}$.

**Lemma**

Let $\phi$ be a 2nd-order sentence. Then $\phi$ is Boolean-valid iff for any set forcing $\mathbb{P}$ and any $\mathbb{P}$-generic filter $G$ over $V$, $\phi$ is valid via full semantics in $V[G]$. 
Result 1; Validity

Theorem
If $\Omega$-conjecture is true and there is a proper class of Woodin cardinals, then $0^{2^b}$ is $\Delta_2$ in set theory.

Remark
The validity of full second order logic is $\Pi_2$-complete in ZFC.
Ω-logic; Ω-validity

Ω-logic: a logic of forcing absoluteness

**Definition (Ω-validity)**

Let $\phi$ be a $\Pi_2$-sentence in set theory. Then $\phi$ is $\Omega$-valid if $\phi$ is true in any set forcing extension.

Main interest: $0^\Omega = \{\phi \mid \phi$ is $\Omega$-valid\}.
Definition

A set of reals $A$ is **universally Baire** if for any continuous function $f$ from a compact Hausdorff space $X$ to the reals, $f^{-1}(A)$ has the property of Baire in $X$. 
Ω-logic; Universally Baire sets

**Definition**

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**Remark**

A set of reals $A$ is universally Baire if and only if for any partial order $\mathbb{P}$, there are trees $T, U$ on $\omega \times Y$ for some $Y$ such that

\[ A = p[T] \text{ and } \Vdash_{\mathbb{P}} \text{“} p[\tilde{T}] = \mathbb{R} \setminus p[\tilde{U}] \text{”}. \]
## Definition

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## Example

1. The collection of all uB sets is closed under complements and countable unions, hence every Borel set is universally Baire.
**Ω-logic; Universally Baire sets**

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**Example**
1. The collection of all uB sets is closed under complements and countable unions, hence every Borel set is universally Baire.
2. Every $\mathcal{P}_{1}^1$-set of reals is universally Baire.
Ω-logic; Universally Baire sets ctd.

Example

The following are equivalent:

1. every \( \Pi^1_2 \)-set of reals is universally Baire,
2. every set has a sharp.
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   1. every $\Pi^1_2$-set of reals is universally Baire,
   2. every set has a sharp.

2. The following are equivalent:
   1. every set of reals in $L(\mathbb{R})$ is universally Baire,
   2. for any set $X$, $M^\#_\omega(X)$ exists.

\(\Omega\)-logic; Universally Baire sets ctd.
Ω-logic; Closure under universally Baire sets

**Definition (A-closure)**

Let $A$ be universally Baire. A countable $\omega$-model $M$ of ZFC is $A$-closed if for any $M$-generic filter $G$ on a partial order in $M$,

$$M[G] \cap A \in M[G].$$
Ω-logic; Closure under universally Baire sets

**Definition (A-closure)**

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**Example**

- For a countable $\omega$-model $M$ of ZFC, the following are equivalent:
  - $M$ is $A$-closed for any $\Pi^1_1$-set $A$, and
  - $M$ is well-founded.
Definition (A-closure)

Let $A$ be universally Baire. A countable $\omega$-model $M$ of ZFC is $A$-closed if for any $M$-generic filter $G$ on a partial order in $M$,

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Example

1. For a countable $\omega$-model $M$ of ZFC, the following are equivalent:
   - $M$ is $A$-closed for any $\Pi_1^1$-set $A$, and
   - $M$ is well-founded.

2. For a countable $\omega$-model $M$ of ZFC, the following are equivalent:
   1. $M$ is $A$-closed for every $\Pi_2^1$-set $A$, and
   2. $M$ is closed under sharps.
\(\Omega\)-logic; \(\Omega\)-provability

**Definition**

Let \(\phi\) be a \(\Pi_2\)-sentence in set theory. Then \(\phi\) is \(\Omega\)-provable if there is a universally Baire set \(A\) such that

\[
(\forall M \text{ c.t.m. of ZFC}) \text{ if } M \text{ is } A\text{-closed, then } M \models \phi.
\]
**Definition**

Let $\phi$ be a $\Pi^1_2$-sentence in set theory. Then $\phi$ is $\Omega$-provable if there is a universally Baire set $A$ such that

$$(\forall M \text{ c.t.m. of ZFC}) \text{ if } M \text{ is } A\text{-closed, then } M \models \phi.$$ 

**Example**

If every set has a sharp, any $\Pi^1_3$-sentence true in $V$ is $\Omega$-provable.
Theorem (Soundness (Woodin))

Let $\phi$ be a $\Pi_2$-sentence. Then $\phi$ is $\Omega$-provable, then it is $\Omega$-valid.
**Theorem (Soundness (Woodin))**

Let $\phi$ be a $\Pi_2$-sentence. Then $\phi$ is $\Omega$-provable, then it is $\Omega$-valid.

**Conjecture (\(\Omega\)-conjecture (Woodin))**

Suppose there is a proper class of Woodin cardinals and let $\phi$ be a $\Pi_2$-sentence. Then $\phi$ is $\Omega$-provable iff $\phi$ is $\Omega$-valid.
**Theorem (Soundness (Woodin))**

Let \( \phi \) be a \( \Pi_2 \)-sentence. Then \( \phi \) is \( \Omega \)-provable, then it is \( \Omega \)-valid.

**Conjecture (\( \Omega \)-conjecture (Woodin))**

Suppose there is a proper class of Woodin cardinals and let \( \phi \) be a \( \Pi_2 \)-sentence. Then \( \phi \) is \( \Omega \)-provable iff \( \phi \) is \( \Omega \)-valid.

**Theorem (Woodin)**

\( \text{ZFC} + \Omega \)-conjecture + “There is a proper class of Woodin cardinals” is consistent.
Result 1; Validity

Theorem
If \( \Omega \)-conjecture is true and there is a proper class of Woodin cardinals, then \( 0^{2^b} \) is \( \Delta_2 \) in set theory.
Result 1; Validity

Theorem
If $\Omega$-conjecture is true and there is a proper class of Woodin cardinals, then $0^{2^b}$ is $\Delta_2$ in set theory.

Theorem
$$0^\Omega \equiv_T 0^{2^b}.$$ 

Theorem (Woodin)
Assuming $\Omega$-conjecture and a proper class of Woodins, one can show that $0^\Omega$ is $\Delta_2$ in Set Theory.
Result 1; Validity ctd.

**Theorem**

\[ 0^\Omega \equiv_T 0^{2^b}. \]

**Key point:**

**Lemma**

Let \( \phi \) be a 2nd-order sentence. Then \( \phi \) is Boolean-valid iff for any set forcing \( \mathbb{P} \) and any \( \mathbb{P} \)-generic filter \( G \) over \( V \), \( \phi \) is valid via full semantics in \( V[G] \).
Result 2; Compactness numbers

**Definition**

$k$ is *strongly compact* if $k$ is $L^1_{k,k}$-compact.
Result 2; Compactness numbers

**Definition**

\( \kappa \) is *strongly compact* if \( \kappa \) is \( \mathbb{L}^{1}_{\kappa, \kappa} \)-compact.

**Theorem (Magidor)**

The following are equivalent:

1. \( \kappa \) is \( \mathbb{L}^{2}_{\kappa, \kappa} \)-compact,
2. \( \kappa \) is extendible.
Result 2; Compactness numbers

**Definition**

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**Theorem (Magidor)**

The following are equivalent:

1. \( \kappa \) is \( \mathbb{L}^2_{\kappa, \kappa} \)-compact,
2. \( \kappa \) is extendible.

**Theorem**

Suppose there is a proper class of Woodin cardinals, a supercompact cardinal \( \kappa \), and assume Strong \( \Omega \)-conjecture holds. Then \( \kappa \) is \( \mathbb{L}^2_{\kappa, \kappa} \)-compact.

**Definition (Strong \( \Omega \)-conjecture)**

Assume there is a proper class of Woodin cardinals. Then \( \Omega \)-conjecture with real parameters holds in any set generic extension.
Löwenheim-Skolem number

Definition

The **cardinality** of a structure $M$ ($\text{card}(M)$) is that of its first order part.
Löwenheim-Skolem number

**Definition**

The **cardinality** of a structure $M$ ($\text{card}(M)$) is that of its first order part.

**Definition**

Given a logic $L$, the **Löwenheim-Skolem number** of $L$ ($\ell(L)$) is the least $\kappa$ such that

$$(\forall \phi \in L) \ (\exists M) \ M \models \phi \iff (\exists M) \ M \models \phi \text{ and } \text{card}(M) \leq \kappa.$$
Löwenheim-Skolem number

Definition

The cardinality of a structure $M$ (card($M$)) is that of its first order part.

Definition

Given a logic $L$, the Löwenheim-Skolem number of $L$ ($\ell(L)$) is the least $\kappa$ such that

$$(\forall \phi \in L) \ (\exists M) \ M \vDash \phi \implies (\exists M) \ M \vDash \phi \text{ and card}(M) \leq \kappa.$$ 

Example

1. $\ell(\text{FOL}) = \aleph_0$
Löwenheim-Skolem number

Definition

The **cardinality** of a structure $M$ (card$(M)$) is that of its first order part.

Definition

Given a logic $L$, the **Löwenheim-Skolem number** of $L$ (Γ$(L)$) is the least $\kappa$ such that

$$(\forall \phi \in L) \ (\exists M) \ M \models \phi \implies (\exists M) \ M \models \phi \text{ and card}(M) \leq \kappa.$$  

Example

1. $\ell$ (FOL) = $\aleph_0$
2. $\ell$ (full SOL) = sup{$\alpha \mid \alpha$ is $\Delta_2$-definable}. So

   (The first Woodin limit of Woodins) < $\ell$ (full SOL) ≤ (The first $\Sigma_2$ reflecting card).
Result 3; Löwenheim-Skolem number

Example

1. $\ell$ (FOL) = $\kappa_0$

2. $\ell$ (full SOL) = $\sup\{\alpha | \alpha$ is $\Delta_2$-definable}. So

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   $\leq$ (The first $\Sigma_2$ reflecting card).
Result 3; Löwenheim-Skolem number

Example

1. $\ell$ (FOL) = $\aleph_0$
2. $\ell$ (full SOL) = $\sup\{\alpha \mid \alpha \text{ is } \Delta_2\text{-definable}\}$. So

   (The first Woodin limit of Woodins) $< \ell$ (full SOL)

   \[\leq (\text{The first } \Sigma_2 \text{ reflecting card}).\]

Theorem

If ZFC + “There is a proper class of Woodin cardinals” is consistent, then so is ZFC + “There is a proper class of Woodin cardinals” + “$\ell$ (BVSOL) $< (\text{the first Woodin cardinal})$”
Result 4; Completeness

One can formulate the notion of provability in BVSOL (Boolean provability) in a similar way as $\Omega$-provability.
Result 4; Completeness

One can formulate the notion of provability in BVSOL (Boolean provability) in a similar way as Ω-provability.

Theorem (Soundness)

If φ is Boolean provable, then it is Boolean valid.
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One can formulate the notion of provability in BVSOL (Boolean provability) in a similar way as Ω-provability.

**Theorem (Soundness)**

If φ is Boolean provable, then it is Boolean valid.

**Definition (Completeness)**

Completeness of BVSOL states the following: Assume there is a proper class of Woodin cardinals. Then if φ is Boolean valid, then so is Boolean provable.
Result 4; Completeness

One can formulate the notion of provability in BVSOL (Boolean provability) in a similar way as $\Omega$-provability.

**Theorem (Soundness)**

If $\phi$ is Boolean provable, then it is Boolean valid.

**Definition (Completeness)**

Completeness of BVSOL states the following: Assume there is a proper class of Woodin cardinals. Then if $\phi$ is Boolean valid, then so is Boolean provable.

**Theorem**

Completeness of BVSOL implies $\Omega$-conjecture.

Note: The converse is not known to be true.
Inner models from logic

Definition

Given a logic $\mathcal{L}$,

\[
\begin{align*}
L_0(\mathcal{L}) &= \emptyset, \\
L_{\alpha+1}(\mathcal{L}) &= \text{Def}_\mathcal{L}(L_\alpha(\mathcal{L})), \\
L_\gamma(\mathcal{L}) &= \bigcup_{\alpha<\gamma} L_\alpha(\mathcal{L}) \quad (\gamma \text{ is limit}), \\
L(\mathcal{L}) &= \bigcup_{\alpha \in \text{On}} L_\alpha(\mathcal{L}).
\end{align*}
\]
Inner models from logic

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\end{align*}
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**Example**

- When $\mathcal{L}$ is FOL, $L(\mathcal{L})$ is $L$.
- When $\mathcal{L}$ is full SOL, $L(\mathcal{L})$ is HOD.
Example (Kennedy, Magidor, Väänänen)

Let $\mathcal{L}$ be the countable cofinality logic. Then assuming a proper class of Woodins, the theory of $\mathcal{L}$ is invariant under set forcing, $\mathcal{L}$ contains $0^\dagger$, and $\mathcal{L}$ is not $\Sigma^1_3$-correct.

Question

How about $\mathcal{L} = \text{BVSOL}$?
$L(\mathcal{L})$ for BVSOL

**Definition**

For a set $X$ and a second order formula $\phi$,

$$A_{\phi, X} = \{x \in X \mid \phi[x] \text{ is Boolean valid with the first order universe } X\}.$$ 

$$\text{Def}_{\text{BVSOL}}(X) = \{A_{\phi, X} \mid \phi \text{ is a second order formula}\}.$$ 

Let $L^* = L(\text{BVSOL})$. 
$L(\mathcal{L})$ for BVSOL, ctd.

**Proposition (Schlicht, I.)**

$L^*$ can express the continuum function. Therefore, one can force $V = L^*$ with a class forcing.
\( L(\mathcal{L}) \) for BVSOL, ctd.

**Proposition (Schlicht, I.)**

\( L^* \) can express the continuum function. Therefore, one can force \( V = L^* \) with a class forcing.

**Question**

Can one force \( L^* \neq \text{HOD} \)?
**Definition**

For a set $X$ and a second order formula $\phi$, $\phi$ is **suitable to** $X$ if for every element $x$ of $X$, either $\phi[x]$ or $\neg\phi[x]$ is Boolean valid with the first order universe $X$. 
$L(\mathcal{L})$ for BVSOL, ctd..

**Definition**

For a set $X$ and a second order formula $\phi$, $
\phi$ is **suitable to** $X$ if for every element $x$ of $X$, either $\phi[x]$ or $\neg \phi[x]$ is Boolean valid with the first order universe $X$.

**Definition**

$$\text{Def}_{\text{BVSOL}}^{ab}(X) = \{ A_{\phi, x} \mid \phi \text{ is suitable to } X \}.$$  

Let $L_{ab}^* = L(\text{BVSOL})$ with $\text{Def}^{ab}$. 
Proposition (Schlicht, I.)

$L^*_a b$ is $\Sigma^1_\omega$-correct. In particular, projective determinacy is true in $L^*_a b$ assuming large cardinals.
L(\mathcal{L}) for BVSOL, ctd...

Proposition (Schlicht, I.)

$L^*_{ab}$ is $\Sigma^1_\omega$-correct. In particular, projective determinacy is true in $L^*_{ab}$ assuming large cardinals.

Question

Could AD$^{L(\mathbb{R})}$ be true in $L^*_{ab}$?
$L(\mathcal{L})$ for BVSOL, ctd...

Proposition (Schlicht, I.)
$L_{ab}^*$ is $\Sigma^1_1$-correct. In particular, projective determinacy is true in $L_{ab}^*$ assuming large cardinals.

Question
Could $AD^{L(\mathbb{R})}$ be true in $L_{ab}^*$?

Question
Is $L_{ab}^*$ absolute under set forcing? How about its theory?
$L(\mathcal{L})$ for BVSOL, ctd...

**Proposition (Schlicht, I.)**

$L^*_a_b$ is $\Sigma^1_\omega$-correct. In particular, projective determinacy is true in $L^*_a_b$ assuming large cardinals.

**Question**

Could $AD^{L(\mathbb{R})}$ be true in $L^*_a_b$?

**Question**

Is $L^*_a_b$ absolute under set forcing? How about its theory?

**Proposition (Schlicht, I.)**

$(L^*_a_b)^{V^\mathbb{P}} \subseteq \text{HOD}^V$ for all set forcings $\mathbb{P}$. 
Vielen Dank für Ihre Aufmerksamkeit!