Abstract—In this paper, we introduce altitude control of coaxial-helicopter. Unmanned aerial vehicle (UAV) is widely used with high performance computer (HPC). Additionally, flight control used for a UAV with advanced mathematics. However, HPC has several problems, such as increase of cost and processing delays in calculation. We can solve the problem if be same control can do with a low performance computer, hence miniaturization of a UAV is achieved. For this reason, we apply limited pole placement (LPP) controller design on a helicopter, which is capable to find out critical efficiency of controller and time delay compensation. Finally, in the paper presents the validation results from the altitude control on a coaxial helicopter UAV.

I. INTRODUCTION

UAV is required to perform precision control with various sensors [1]. Then we often use HPC systems. However, it increases cost and weight up. If the same control performance can be achieved by low performance computer, miniaturization, weight saving, and mass production of UAV are performed.

In practice, controller the UAV is designed a discrete time system high precision use discrete time control a UAV, nevertheless various time delay elements in the control system. These time delay elements lead to phase lag, which causes degrading control performance. In the present of UAV, which is refer to energetically made about these problems. Ohara has reported that compensation of time delay by a servo system using state prediction [2]. Dydek and his coworkers have proposed the new adaptive control to time delay [3]. By the conventional technique, such as the including time delay in a model by approximation theories, adjustment of a controller parameter, an application of time delay compensation control system state prediction control, robust control theory, etc. has been considered. However, on the basis of approximation theory leads to an increase in a degree of control system. Furthermore, these are a problem to which only low frequency range is approximated. In order to rely on the experience of designers by adjustment of parameters of the controller, which is forecast for the design of the controller is bad in the report Shigemasa [4]. Thus, a process control of a large time delay is assumed in a time delay control represented by stating prediction control, which is not suitable for minute useless time control of operation processing delay, etc. Hence, we think application of the LPP which Urakawa proposes is considered. The LPP is a parameter derivation of the controller for arranging a pole equivalent to the number of the parameter controller, and even when using fixed composition controllers, such as PID control which can obtain the parameter in consideration of time delay. Furthermore, the new consideration of time delay was attained in analyzing an action in placement pole and determined pole. In this present, the parameter which takes the influence of time delay into consideration by using the LPP to propose of realizing altitude control on the coaxial helicopter is calculated. In addition to the effect is verified.

In the next section, the model of the control object is introduced. In Section II, influence of calculation time delay. In estimated, experimental results of the altitude control of the coaxial helicopter are presented in Section IV.

II. INFLUENCE OF TIME DELAY

A. Modeling

The equation of motion of a coaxial helicopter. In reported in the past [6] (see Fig.2). We find for the electric basic equation of DC motor specifically carried in a coaxial helicopter and the mechanical point of equation of motion, the equation of motion in represented an rotor blade [8].

\[ V_m = R_m i_m + K_E \omega_m \]  
\[ \tau_m = K_T i_m \]  
\[ \tau_m = J_m \frac{d\omega_m}{dt} + C_m \omega_m + \tau_l \]  
\[ n\tau_l = I_r \frac{d\omega_r}{dt} + C_r \omega_r. \]

Table I shows all of variable and sign which are used in the formula. The equation of motion (4) of the blade connected as
a load is unified to the equation of motion (3) of DC motor.

\[ \tau_m = J \frac{d\omega_m}{dt} + C \omega_m + \alpha \]  \hspace{1cm} (5)

where, \( J = J_m + \frac{\rho A}{2} \), \( C = C_m + \frac{\eta}{2} \) express the moment of inertia after integration, and viscous friction. DC motor voltage \( V_m \) and the relation of the angular velocity \( \omega_m \) of a formula (5) are changed to transfer function expression.

\[ \frac{\omega_m(s)}{V_m(s)} = \frac{K_t}{R_m (J s^2 + C s + K_E^2)} . \]  \hspace{1cm} (6)

Meanwhile, lift which the rotor blade of a helicopter generates is called for based on blade element theory [9].

\[ T = \frac{b}{4} \rho m \omega_m^2 R^3 (\theta + \phi_t) c = K \omega_m^2 \]  \hspace{1cm} (7)

thus, there is little influence which has on a lift as compared with angular velocity \( \omega_m \), the inflow \( \phi \) of a blade tip disregarded now. Perpendicular movement of a helicopter can be similarly realized to be a uniform movement of mass point. The relation between advanced direction \( Z \) and lift \( T \) is expressed by the Lagrange equation of motion.

\[ m \ddot{z}(t) = T - mg. \]  \hspace{1cm} (8)

Thus, the lift of a helicopter is proportional to the square of angular velocity, in order to be linearised a control model. It decomposes into altitude control system and angular velocity control system that consider it [7]. Lift \( T \) considers gravity to \( U = t - mg \) to control the input \( U \) because it cannot take a negative value. Where suppose that \( T > mg \) is always realized. The helicopter is controlled by controlling input \( U \) and angular velocity is controlled by calculating a speed instruction value from the following formulas.

\[ \omega = \sqrt{\frac{u + mg}{K}}. \]  \hspace{1cm} (9)

(7) thus, it linearized to the formula, a control model can be expressed as a transfer function. A control input is set \( U(s) \) and advanced displacement magnitude is set to \( Z(s) \). A control model will be obtained if the Laplace transform is applied to the equation of motion of the helicopter.

\[ G(s) = \frac{Z(s)}{U(s)} = \frac{1}{ms^2} . \]  \hspace{1cm} (10)

**B. System identification**

System identification is used in order to find for the parameter of DC motor. The high order ARX model is used and it finds for a parameter [10]. Make an incoming signal into the voltage of DC motor, and let an output signal is the angular velocity of DC motor. A signal inputs M-sequence and it finds for a parameter [10]. Make an incoming signal parameter of DC motor. The high order ARX model is used and it finds for a parameter [10]. Make an incoming signal into the voltage of DC motor, and let an output signal is the angular velocity of DC motor.

\[ \omega_m(s) = \frac{\alpha}{s^2 + \beta} = \frac{11.66}{s + 9.192} \cdot exp(-0.03s) \]  \hspace{1cm} (11)

where, \( \alpha = \frac{K_t}{J_m} \), \( \beta = \left( C + K_E^2 \right) \cdot \frac{\rho A}{2} \). The impulse response was inputted to the DC motor. Time delay to 30 milliseconds was checked at a response time. The controller which compensates found process hold-up time with the LPP introduced in the following chapter is built.

**C. Time delay influence of continous system**

The influence of time delay is evaluated in the controlled object expressed by a discrete system. Fig.3 shows using the
TABLE II. CONTROL PARAMETERS FOR CONTINUOUS MODEL

<table>
<thead>
<tr>
<th></th>
<th>0.99978</th>
<th>0.99935</th>
<th>0.99892</th>
<th>0.9956</th>
<th>0.987</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>0.536</td>
<td>127.818</td>
<td>1613.59</td>
<td>1.45e06</td>
<td>2.67e08</td>
</tr>
<tr>
<td>$K_i$</td>
<td>0.2476</td>
<td>0.728</td>
<td>1.192</td>
<td>4.193</td>
<td>9.512</td>
</tr>
<tr>
<td>$K_d$</td>
<td>24.269</td>
<td>0.9529</td>
<td>0.239</td>
<td>0.042</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Closed-loop transfer function of a continuous system is created. Furthermore, we finds for pole and zero by coefficient comparison of transfer function. Simultaneous equations were used in order to find for a parameter.

$$G_{ty}(s) = \frac{C(s)P(s)}{1 + C(s)P(s)}$$
$$= \frac{K_p \left(1 + K_ds + K_i \frac{1}{s} + K_p \frac{1}{s^2} + K_d \frac{1}{s^3}\right)}{1 + K_p \left(1 + K_ds + K_i \frac{1}{s} + K_p \frac{1}{s^2} + K_d \frac{1}{s^3}\right)}$$

where $A$, $B$, $G$, $P$, and $Z$ are coefficients of numerator denominator polynomial. Define $D = e^{-t}$ as a coefficient. Fig.3 shows a pole placement is used based on the continuous system model. As a result, a controller parameter is estimated.

Fig. 5. Evaluation of the delay time by the PID controller

Fig. 6. Digital control system including a delay time

A. Design of LPP

The LPP is applied to a controlled object. A controller uses PID control. A PID controller is first divided into a controllable parameter part to find a parameter, and a fixity coefficient part. Namely, the portion which is not related to derivation of $K_p$, $K_i$, and $K_d$ which are controllable parameters is extracted. Therefore, a controlled object does not change a parameter, it
combines with a fixity coefficient part.

\[
K(z) = \frac{K_{np}(z)}{K_{dp}(z)} \cdot \frac{K_{nf}(z)}{K_{df}(z)} = \left( \frac{K_p(z+1) + K_{dP}(z-1)^2 + K_{dP}z(z+1)}{z(z-1)} \right) \frac{1}{z-1}
\]

(17)

\[P(z) = P_n(z)\]

\[\frac{K_{dp}(z)}{K_{df}(z)} \cdot \frac{P_d(z)}{d(z)} = \frac{z^2 + b_1 z + b_0}{z^5 + a_7 z^4 + a_6 z^3 + a_5 z^2 + a_4 z + \alpha_0} \]

(19)

where, \(N(z)\) and \(D(z)\) are rational functions of fixity coefficient. \(\alpha(z)\) and \(\beta(z)\) are rational functions of parametric part. Degree of numerator denominator of (19) and (20) types shows the number of poles a control system. The relation between the placement pole to LPP and determined pole is denoted by the following formula. Number of determining pole is eight. Number of placement pole is three of a controllable parameter and the same number.

\[n_q = n_a - n_a - n_q - 1 = 8 - 0 - 2 - 1 = 5\]

(21)

where, \(n_a\) and \(n_q\) are the number of the denominator coefficients of a closed-loop transfer function and number of determined pole, \(n_a\) and \(n_q\) are the number of the coefficients of the rational function of a parameter part. placement pole is set to \(p\) and determined pole is set to \(q\). Furthermore, Fig. 7 shows the closed-loop transfer function of discrete system is created. A denominator polynomial is set to \(\gamma(z)\), and a characteristic polynomial is drawn.

\[
\gamma(z) = P_p(z) (z_a^n + Q_p(z)) = \prod_{i=1}^{n_a} (z - p_i) \cdot \prod_{j=1}^{n_q} (z - q_j)
\]

(22)

where, \(P_p(j = 0, 1, 2)\) and \(Q_p(k = 0, 1, 2, 3, 4)\) are coefficients when it develops to a polynomial. This is like desired positioning poles have closed-loop transfer function parameter, and same as to dependent determined poles. Where, placement pole is a specified pole. A determined pole is a pole determined automatically other than the placement pole. Then, a pole assignment is performed using the LPP. The characteristic polynomial of the closed-loop transfer function of Fig. 6 is denoted by a Diophantine equation as well as polynomial algebraic [11].

\[
P_p(z)z_a^n = \alpha(z)d(z) + \beta(z)n(z) - Q_p(z)P_p(z).
\]

(23)

We find for the fixity coefficient part denoted by (19) equation as a matrix. Where, let \(\theta\) be a parametric part and a coefficient vector of a determined pole. Let \(\psi\) be a coefficient vector of an arrangement pole. The formula which finds for a parametric part:

\[
\theta^T = \psi^T E^{-1}
\]

(24)

\[
\theta^T = \begin{bmatrix} \alpha_0 & \beta_0 & \beta_1 & \beta_2 & Q_0 & Q_1 & Q_2 & Q_3 & Q_4 \end{bmatrix}
\]

(25)

\[
\psi^T = [0 0 0 0 0 \theta_0 P_1 P_2 1]
\]

(26)

\[
E = \begin{bmatrix} 0 & 0 & 0 & 0 & a_4 & a_5 & a_6 & a_7 & 1 \\ b_0 & b_1 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_0 & b_1 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -P_0 & P_1 & P_0 & -P_2 & P_1 & P_2 & 0 \\ 0 & 0 & -P_0 & -P_1 & -P_0 & P_2 & -P_1 & -P_2 & 0 \\ 0 & 0 & 0 & -P_0 & -P_1 & -P_2 & P_0 & -P_1 & -P_2 \end{bmatrix}
\]

(27)

The coefficient of parameter part can be obtained from this. In addition, \(K_P, K_i,\) and \(K_d\) are calculated from the equation (20).

B. Simulation

The effect of the LPP is checked. It arranges to same pole as the simulation of continuous system performed as the point. Specifically, a pole is arranged in the same position 0.99978 (about -1 rad/s), 0.99935 (about -3 rad/s), 0.99892 (about -5 rad/s), 0.9956 (about -20 rad/s), and 0.9870 (about -60 rad/s). Table III shows controller parameter corresponding to each pole. The response of discrete system model with time delay. Using these parameters. Response comparison of desired value and LPP was performed (see Fig. 8). In 0.99978 (about -1 rad/s) which is a late pole assignment, convergence of the LPP has become slow. When the parameter of continuous system is compared with the parameter of the LPP, there is a difference among about ten times. In the case of 0.99935 (about -3 rad/s), the response of a continuous system is fast. However, in the LPP which not become a vibration response. Although it has influence of the phase lag by time delay, it is domain which can realize the response that is not vibration by adjustment of a controllable parameter. In the case of 0.99892 (about -5 rad/s), result to vibration is brought by continuous system. In contrast, in the LPP, it does not become in vibration. In addition, speed to converge is improving. The case of 0.9956 (about -20 rad/s), as for the LPP, overshooting is reduced comparatively. Furthermore, speed to converge is improving. When a controllable parameter is compared, it turned out that the influence of the phase lag by time delay is compensated by the differentiation ingredient. Finally, by the case of 0.9870 (about -60 rad/s), overshooting is further suppressed and flat nature of desired value is improving.

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>CONTROL PARAMETERS FOR DISCRETE MODEL AND DETERMINED POLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_p)</td>
<td>0.0167</td>
</tr>
<tr>
<td>(K_i)</td>
<td>0.0073</td>
</tr>
<tr>
<td>(K_d)</td>
<td>45.5572</td>
</tr>
<tr>
<td>(q_1)</td>
<td>0.94819</td>
</tr>
<tr>
<td>(q_2, q_5)</td>
<td>-3.3(\times)</td>
</tr>
<tr>
<td>(q_3)</td>
<td>-0.0863</td>
</tr>
<tr>
<td>(q_4)</td>
<td>0.0929</td>
</tr>
</tbody>
</table>
Fig. 8. Limited pole placement method using the PID controller

Fig. 9. Position of the determined poles

The parameter of controller became unstable when the pole had been arranged in the fast position than 0.9870 (about -60 rad/s).

C. Analysis of pole

Analysis of the placement pole and determined pole which were determined with the LPP is conducted (see Fig.9). The spatial relationship of the placement pole and determined pole which have been arranged by PID control is considered (see Fig.10). Where, the placement pole is ○ mark. A determined pole is denoted by ✗ mark. In the case of 0.99978 (about -1 rad/s), a determined pole is located in a fast position than placement pole. However, when a placement pole moves to a fast position, it turns out that a determined pole is moved into late position. In the case of 0.9870 (about -60 rad/s), the position of placement pole and determined pole turns into the almost same position. Furthermore, if it becomes a fast position about a placement pole, a determined pole will come out of a unit circle. Namely, a control system becomes unstable and a response deteriorates (see Fig.10 (e)). The performance limits of altitude control of the helicopter by discrete system PID control will be about 0.9870 (about -60 rad/s). Furthermore, even if it’s going to realize a fast response, a determined pole will become later than placement pole. Hence, shows that it is unreliable.

IV. EXPERIMENT

The experiment was conducted using simulation results. Fig.11 and Fig.12 shows the equipment used for the experiment. It is equipment which carries out the front and rear, right and left restraint of the movement of the helicopter. The experiment was conducted using the controllable parameter of performance limit. Fig.13 shows the obtained helicopter trajectories (height). It is converged by the response error at about ±10 centimeter to the desired value. Although the equipment for restraining movement of helicopter was used, body balance sometimes collapsed a little. However, body by manual operation was stabilized. A wavelike vibration is the
Influence. In addition, the orbital function was used for the control input in order to take the saturation characteristic of a helicopter into consideration. Stand up of the helicopter is late from reason.

V. CONCLUSION

In this paper, solutions using the LPP were shown on the altitude control of the coaxial helicopter including time delay by arithmetic processing. In addition, validity was checked by conducting a simulation and a systematic experiment. The above results showed that the parameter design of prospect was made possible. In addition, the performance limit of the controller has been checked. In consequence, the boundary of performance limit was checked by grasping locus a pole.

REFERENCES