

# Equilibrium Motion Planning of Humanoid Climbing Robot under Constraints

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This paper introduces an equilibrium position planning on humanoid climbing robot. Recently, free-climbing has become popular and many climbing gyms have been opened. On the other hand, robots are expected as useful machines to assist human lives. To develop climbing robot is to expand the ability of humanoid robots and they can be used at not only horizontal spaces but also vertical space in mountain areas or disaster areas. However, we need to create variety of techniques to develop such the humanoid climbing robots. This paper focus on the method to derive optimal position of the center of mass of the robot under physical constraints making use of optimization theory.

**Keywords:** Humanoid robot, climbing, planning, control, optimization

## 1. Introduction

Humanoid robots are widely expected to use in daily lives. This paper introduces the usage in vertical world as a solution to extend the ability of humanoid robots<sup>(1)(2)</sup>. Moreover, the study of climbing robots are related to not only new robot creation but also to create new motion control technology, and it is also the study of human being itself. The application of humanoid climbing robots seems to be observation, carriage, and rescue in mountain area or disaster areas. This paper focuses on only the balance control technique considering equilibrium state with nonlinear optimization theory, although we need the variety of climbing technique.

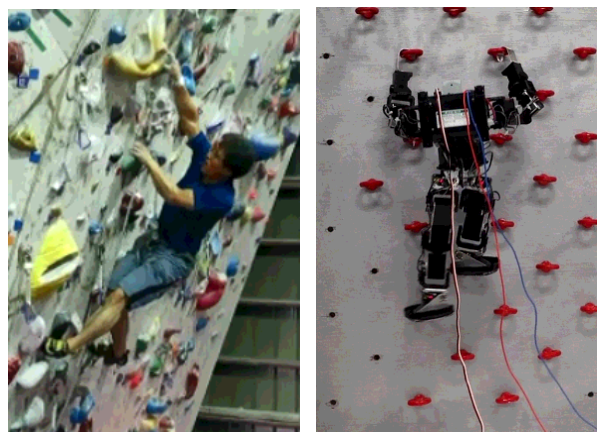
Recently, some humanoid climbing robots have been reported<sup>(3)(2)</sup>, but they has never introduced the motion but has introduced the extended application with respect to planning considering collision avoidance. Dung et.al. introduced a planning method with pre-observation using Kinect sensor, and a modeling method using MATLAB<sup>(5)(6)</sup>. Moreover, they introduce a balance control algorithm for the simplest model but it was not general.

In the case of climbing, the constraints consists of reachability, structural solvability, force direction, holding power, equilibrium of force and moment, joint torques, etc. in 3D space. These constraints can be expressed as equations or inequations. Generally, autonomous robots including humanoid robots are expected to minimize the used energy under such the constraints. For such case, optimizatio theory seems to be useful.

Hence this paper presents a climbing planning algorithm considering the practical constraints in 3D space by making use of optimization theory.

## 2. Principle of Climbing and Constraints

### 2.1 Climbing Climbing ia a sport to climb natural or



(a) Real climbing by Author

(b) Climbing robot

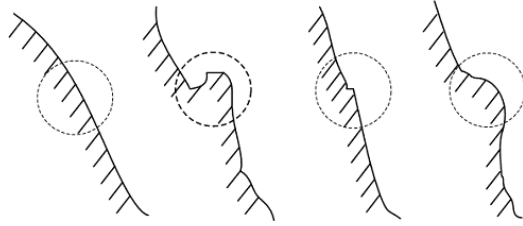
Fig. 1. Climbing examples.

artificial wall by using four limbs. Fig.1 illustrates an example of climbing by an author and humanoid robot<sup>(2)</sup>. The climbing principle is as the followings. The humanoid robot is expected to keep balance of the body inside of reachable region under the gravity force. Basically, it repeats the motions to move one limb to next hold, and it move from the starting position to the end position. Suppose that a robot has  $N$  limbs and put  $n(\leq N)$  limbs on some holds. Here, the word ‘put’ means a generic term to represent grasping holds, hooking fingers on holds, pushing holds by palms, and riding holds on toes or heels. They generate forces to keep equilibrium state of robot.

However, there are limits of hands or feet forces to keep balance. Their reaction forces are generated as the forces given to hands and feet with oposite directions on holds. Where the joints of robots consist of rotational joints. Since the arm forces and legs forces to keep balance are generated by joint torques, the maximum torques should be considered to perform them. The protrusions or holes on wall are called ‘holds’. Robot puts their hand or feet on the holds in order to keep balance. There are varieties of shapes of holds ex-

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(a)Frictionless (b)Bracket (c)Crimp (d)Sloper

Fig. 2. Shapes of climbing holds

pressed by Fig.2. There are some cases of hooking, pressing, and method making use of friction.

**2.2 Consideration of Constraints** We summarize the constraints for the equilibrium state realization of the climbing robot.

- 1) Reachability: Limbs can reach the target holds.
- 2) Structural solvability (inverse kinematics problem): Hands and feet can be placed on the holds
- 3) Force direction: Force vectors are inside of the hold cones.
- 4) Holding power: Holding forces are smaller than the maximum forces.
- 5) Equilibrium of forces: All of forces and moments must be equilibrium to keep balance.
- 6) Torque constraint: Maximum joint torques are smaller than maximum torques of the robot,  $\tau_{i,j} \leq \tau_{ij,max}$

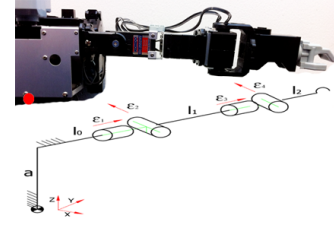
### 3. Structural Solvability

Generally, the shape of humanoid robot is similar to real human. But most of all humanoid robots have more small number of degrees of freedom, although the human have the seven degrees of freedom in the arm from the shoulder to the wrist, and the six degrees of freedom in the leg from the hip joint to the ankle. Therefore, humanoid robot cannot always perform the same pose of real human.

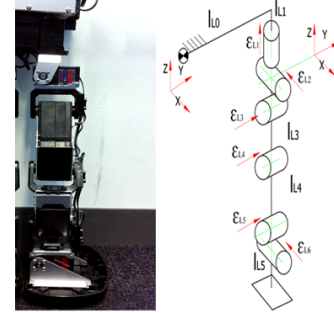
Fig.3 shows an example of the arm and leg of a prototype of humanoid robot. The arm consists of four joints and the leg consists of six joints. Therefore, there is the pose non-available to perform relate to human. The solvability is depend on whether the inverse kinematics problem can be solved or not. It is important problem but this paper omits it.

### 4. Force Direction and Hold Cone

Now, it is assumed that the four limbs are located on their corresponding holds and the balance is kept on. The Fig.4 expresses such the situation. This figure illustrates the sequence via two poses expressing the robot climbing from the lower position to the upper position. The gravity force  $mg$  is given to  $\otimes$  expressing COM and the generated forces by the four limbs are against to the gravity. The force to the feet  $f_{foot}$  is the reaction force by the push force by the foot. When a robot arm pulls a hold with the arm force, the hold gives the climbing force  $f_{hand}$  as the reaction force. Similarly, a foot hold gives another climbing force  $f_{foot}$  as the reaction force of a pushing force by foot. To keep the equilibrium state without fall of the robot, the sum of the forces and the sum of the moments should be kept zero. For instance, the sum of the vertical components of forces cancel out the gravity force.



(a) Structure of the Arm



(b) Structure of the leg

Fig. 3. Structure of arm and leg of the robot

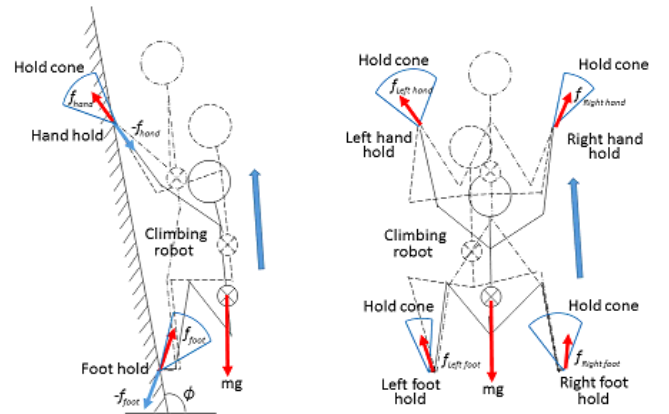


Fig. 4. Shapes of climbing holds

Mentioned above, the shapes of the climbing holds are illustrated in Fig.2. In the case of grasping a bracket holds, the hand can pull the hold for not only vertical downward direction but also obliquely downward direction. Hence, the reaction force direction is not only vertical upward direction but also obliquely upward direction. On the other hand, at a crimp holds, only fingertip or toe can be put on, and the force direction is restricted. In the case of frictionless holds, hand or foot can push to the normal direction of the hold surface. Finally, for sloper holds, hand relies on only friction.

The fan-shapes drawn in Fig.4 express the generable direction of forces, we call them hold-cones. It is similar to friction cone used in robotics field. The center angles of the hold-cone depends on the shape on holds<sup>(8)</sup>.

### 5. Control to Keep Equilibrium State

The force vector  $F_i = [f_{xi}, f_{yi}, f_{zi}]^T (i = 1, 2, \dots, n)$  is defined as the generated force on  $i$ -th hold by Eq.1, which is corresponds to Fig.5.  $f_i$  denotes the absolute value.

$$\begin{aligned} f_{xi} &= f_i \cos \theta_i \cos \phi_i \\ f_{yi} &= f_i \cos \theta_i \sin \phi_i \dots \dots \dots (1) \\ f_{zi} &= f_i \sin \theta_i \end{aligned}$$

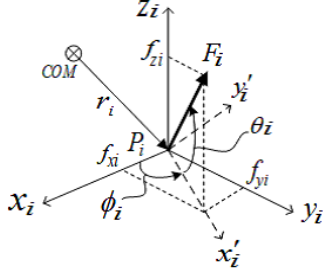


Fig. 5. Definition of force direction

To keep equilibrium state of climbing robot, the sum of total forces and the sum of moments should be zero, respectively. Mentioned above, a robot has  $N$  limbs and put  $n(\leq N)$  limbs on some holds. The vector  $r_i = [r_{xi}, r_{yi}, r_{zi}]^T$  ( $i = 1, 2, \dots, n$ ) is defined as the vector from COM to the  $i$ -th hold. Moreover, it is assumed that any moment is not generated on hold. Then, equilibrium state is expressed as the followings<sup>(8)</sup>.

$$\sum_{i=1}^n f_{xi} = 0 \dots\dots\dots (2)$$

$$\sum_{i=1}^n f_{yi} = 0 \dots\dots\dots (3)$$

$$\sum_{i=1}^n f_{zi} - mg = 0 \dots\dots\dots (4)$$

$$\sum_{i=1}^n r_i \times f_i = \sum_{i=1}^n \hat{r}_i f_i = 0 \dots\dots\dots (5)$$

$$\text{where } n \leq N, \hat{r}_i = \begin{bmatrix} 0 & -r_{zi} & r_{yi} \\ r_{zi} & 0 & -r_{xi} \\ -r_{yi} & r_{xi} & 0 \end{bmatrix}$$

It is summarized as follows.

$$\sum_{i=1}^n (R_i \cdot F_i) - G = 0 \dots\dots\dots (6)$$

where

$$R_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -r_{zi} & r_{yi} \\ r_{zi} & 0 & -r_{xi} \\ -r_{yi} & r_{xi} & 0 \end{bmatrix} \dots\dots\dots (7)$$

The vectors  $F_i = [f_{xi}, f_{yi}, f_{zi}]^T, G = [0, 0, mg, 0, 0, 0]^T$  are the condition vectors to express the equilibrium state.

Moreover, they are converted to the followings.

$$R \cdot F - G = 0 \dots\dots\dots (8)$$

where

$$R = [R_1, R_2, \dots, R_n] \in R^{6 \times 3n},$$

$$F = [F_1^T, F_2^T, \dots, F_n^T]^T \in R^{3n}$$

Generally, the force  $F \in R^{3n}$  to keep the equilibrium state can be derived as the equation (9).

$$F = R^+ \cdot G + (I - R^T R)w \dots\dots\dots (9)$$

However, The  $F$  is not always satisfy the constraints.

## 6. Optimization problem and Constraints

**6.1 Basic Expression and specification** Generally,

optimization problem is known as the problem to minimize  $f(X)$  subject to  $g_i(X) \leq, i = 1, 2, \dots, m$ .

In this paper, we try the optimal position  $P = [x, y, z]^T$  of robot COM to minimize the weighted force function  $f(x) = q_1 f_1^2 + q_2 f_2^2 + \dots + q_n f_n^2, (i = 1, 2, \dots, n)$  under the constraints. Here  $q_i, (i = 1, 2, \dots, n)$  represents the weight value which corresponds to difficulty to keep grasp hand or put foot on the hold.

**6.2 Constraints** It is assumed that the following constraints are given.

- (1) Number of holds:  $n = 3$
- (2) Holds position:  $P_i = [x_i, y_i, z_i]^T$  is given.
- (3) Maximum distances from COM to holds:  $|r_i| \leq r_{i,max}$
- (4) Force directions:  $\theta_i, \phi_i$  are given.
- (5) Maximum forces and min-max position:  $f_i \leq f_{i,max}$
- (6) Sum of components of forces and moments are kept equilibrium, respectively under the gravity force.

To simplify the problem, omit the constraints about the structural solvability and the joint torques of robot in this paper.

In this subsection, the constraints are separated into three parts.

**6.2.1 Linear equation constraint** Here the linear equation constraint is expressed as Eq.(10).

$$A_{eq} X = b_{eq} \dots\dots\dots (10)$$

$$A_{eq} = \begin{bmatrix} \cos \theta_1 \cos \phi_1 & \cos \theta_2 \cos \phi_2 & \cos \theta_3 \cos \phi_3 & 0 & 0 & 0 \\ \cos \theta_1 \sin \phi_1 & \cos \theta_2 \sin \phi_2 & \cos \theta_3 \sin \phi_3 & 0 & 0 & 0 \\ \sin \theta_1 & \sin \theta_2 & \sin \theta_3 & 0 & 0 & 0 \end{bmatrix}$$

$$b_{eq} = \begin{bmatrix} 0 & 0 & mg \end{bmatrix}^T.$$

Where  $X = [f, P]^T, f = [f_1, f_2, f_3]^T, P = [x, y, z]^T$  denotes the magnitude of forces,  $f$  on the hold and COM position,  $P$ . The left three columns of the matrix  $A_{eq}$  and  $B_{eq}$  represent the force constraints corresponding to Eq.(2)-(4).

**6.2.2 Nonlinear constraint** The position of COM is in the reachable region. It is expressed by Eq:(11)-(13).

$$c_1 = \sqrt{r_{x1}^2 + r_{y1}^2 + r_{z1}^2} - r_{1,max} \leq 0 \dots\dots\dots (11)$$

$$c_2 = \sqrt{r_{x2}^2 + r_{y2}^2 + r_{z2}^2} - r_{2,max} \leq 0 \dots\dots\dots (12)$$

$$c_3 = \sqrt{r_{x3}^2 + r_{y3}^2 + r_{z3}^2} - r_{3,max} \leq 0 \dots\dots\dots (13)$$

Where  $r_{x1} = x_1 - x, r_{y1} = y_1 - y, r_{z1} = z_1 - z, r_{x2} = x_2 - x, r_{y2} = y_2 - y, r_{z2} = z_2 - z, r_{x3} = x_3 - x, r_{y3} = y_3 - y, r_{z3} = z_3 - z$ .

The equation constraint  $C_{eq}$  related to the sum of moment is expressed as

$$C_{eq} = r_1 \times F_1 + r_2 \times F_2 + r_3 \times F_3 = 0 \dots\dots\dots (14)$$

Here

$$F_1 = [f_{x1}, f_{y1}, f_{z1}]^T, F_2 = [f_{x2}, f_{y2}, f_{z2}]^T, F_3 = [f_{x3}, f_{y3}, f_{z3}]^T, f_{x1} = f_1 \cos \theta_1 \cos \phi_1, f_{y1} = f_1 \cos \theta_1 \sin \phi_1, f_{z1} = f_1 \sin \theta_1, f_{x2} = f_2 \cos \theta_2 \cos \phi_2, f_{y2} = f_2 \cos \theta_2 \sin \phi_2, f_{z2} = f_2 \sin \theta_2, f_{x3} = f_3 \cos \theta_3 \cos \phi_3, f_{y3} = f_3 \cos \theta_3 \sin \phi_3, f_{z3} = f_3 \sin \theta_3,$$

Eq.(14) corresponds to Eq.(5).

## 7. Numerical simulation

**7.1 Specification for simulation** This section introduces the numerical examples using MATLAB fmincon function. Table.1 shows the specification of the robot. Force  $f_{min}$ ,  $f_{max}$  represents the total finger forces on the holds.

**7.2 Simulation with given force directions** This subsection presents the simulation result to derive optimal force and COM position with three limbs under the fixed force direction. Here it seems that the given force directions are inside of the hold cone.

- (1) Case 1-1 means the simulation result with the basic situation. The values of all of forces equal each others. The optimal position is almost equals the average position.
- (2) Case1-2 shows a simulation result with other weighted  $q_i$ . It means that the hand holds are difficult to grasp or the robot intends to use smaller hand forces(= $f_1, f_2$ ). Then the force on the foot (= $f_3$ ) becomes larger.
- (3) Case 1-3 shows a simulation result on which the hand force directions are different from plumb line.
- (4) Case 1-4 shwos another simulation result on which

Table 1. Specification of Robot

Variables	Symbols	Values	Units
Mass of robot	m	0.60	kg
Gravity axcel.	g	9.8	m/s <sup>2</sup>
Max.dist.	$r_{1,max}, r_{2,max}, r_{3,max}$	0.55, 0.55, 0.65	m
Initial psition	$P_0 = [x_0, y_0, z_0]^T$	Average pos. of holds	m
Minimum position	$P_{min}$	$[-0.1, -0.25, 0.0]^T$	m
Maximum position	$P_{max}$	$[0.20, 0.25, 0.70]^T$	m
Initial force for calc.	$f_0$	$[1.0, 1.0, 1.0]^T$	N
Minimum force on holds	$f_{min}$	$[0.0, 0.0, 0.0]^T$	N
Maximum force on holds	$f_{max}$	$[50, 50, 90]^T$	N

Note: Max.dist. denotes the maximum distance from COM to Hold<sub>*i*</sub>.

Table 2. Position of Holds

Hold	x component	y component	z component
Hold 1	$x_{01} = 0.10$ m	$y_{01} = -0.25$ m	$z_{01} = 0.60$ m
Hold 2	$x_{02} = 0.10$ m	$y_{02} = 0.25$ m	$z_{02} = 0.70$ m
Hold 3	$x_{03} = 0.20$ m	$y_{03} = 0.15$ m	$z_{03} = 0.20$ m

Table 3. Dificulty & Force Directions

Angle	(a) Case1-1	(b) Case 1-2	(c) Case 1-3	(d) Case 1-4
$q_1$	1.0	5.0	1.0	1.0
$q_2$	1.0	5.0	3.0	1.0
$q_3$	1.0	1.0	1.0	1.0
$\phi_1$	180 deg	180 deg	90.0 deg	90.00 deg
$\theta_1$	90.0 deg	90.0 deg	110 deg	100 deg
$\phi_2$	180 deg	180 deg	90.0 deg	90.0 deg
$\theta_2$	90.0 deg	90.0 deg	65.0deg	90.0 deg
$\phi_3$	180 deg	180 deg	90 deg	90.0 deg
$\theta_3$	90.0 deg	90.0 deg	90.0 deg	90.0 deg
$f_1$	1.96 N	0.840 N	2.21 N	0.00 N
$f_2$	1.96 N	0.840 N	1.78 N	2.94 N
$f_3$	1.96 N	4.20 N	2..18 N	2.94 N
$x$	0.13 m	0.17 m	0.14 m	0.15 m
$y$	0.050 m	0.11 m	0.024 m	0.20 m
$z$	0.47 m	0.47 m	0.47 m	0.49 m
$f(X)$	11.5	24.7	12.8	17.3

Note: $q_i(i=1,2,3)$  denotes difficulty of the corresponding hold to keep balance.

$\phi_i(i=1,2,3)$  [deg] denotes force angle about z-axis

and  $\theta_i(i=1,2,3)$  [deg] denotes force angle about y-axis.

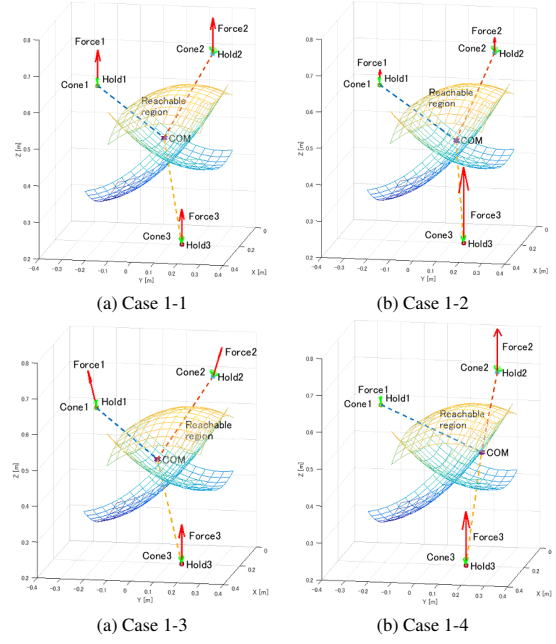


Fig. 6. Simulation example

the force direction of only  $F_1$  is different from plumb line. In this case,  $F_1$  cannot make equivalent condition with other forces. Then the magnitude of  $F_1$  is kept to 0 N. Moreover, the COM position is on the border line of reachable area or out of the area.

## 8. Conclusion

This paper presented the equilibrium motion planning algorithm using nonlinear optimization theory. It seems very useful not only to develop humanoid climbing robot but also to improve real climbing technique of human.

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