Drawing graphs for Fourier series by gnuplot

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In this document I explain how to draw graphs for Fourier series by gnuplot. By Fourier-series-expanding the function

$$f(x) = \begin{cases} 1 & 0 \leq x \leq \pi \\ -1 & -\pi \leq x < 0 \end{cases}$$

on the range $[-\pi, \pi]$ we obtain

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{(2k-1)\pi} \sin((2k-1)x)$$

We can draw a graph for a partial summation up to the n-th term by gnuplot as follows. Firstly we represent the k-th term by a function of two variables x and k as follows.

```
t(x,k) = 4/(pi*(2*k-1))*sin((2*k-1)*x)
```

We can represent the partial sum up to the n-th term by defining a function recursively as follows.

```
series(x,n) = (n>0 ? t(x,n) + series(x,n-1) : 0)
```

The given function $f(x)$ can be written as follows.

```
f(x) = (x>0 ? 1 : -1)
```

By using the functions `series`, `t`, and `f`, we can draw a partial sum up to, for example, the fifth term as follows.

```
set xrange [-pi:pi]
plot series(x,5), f(x)
```

I put the above commands in the file `kukei.txt` and put on my lecture page. We can load the file to the gnuplot by using the redirect as follows.

```
$ gnuplot < kukei.txt
```

Of course you can invoke the gnuplot command and then copy-and-paste the commands in the file `kukei.txt`. If you would like to put the resulting graph in a file, remove the two occurrences of `#` in the file `kukei.txt` (in the case of eps file). You can draw a graph for Fourier series of some other function by changing the definition of the functions `t` and `series` for the case where $n = 0$.

Note The function $f(x)$ and the above series do not have a same value at the points of discontinuity. The relationship between the function $f(x)$ and the Fourier series of $f(x)$ is as follows.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{(2k-1)\pi} \sin((2k-1)x) = \begin{cases} f(x) & 0 < x < \pi \\ 0 & x = -\pi, 0, \pi \\ f(x) & -\pi < x < 0 \end{cases}$$

(This is out of the scope of the class.)