

## Exercise 14-2

Isao Sasano

**Exercise** Calculate the discrete Fourier transform of the following sequence.

$$\mathbf{f} = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \\ 9 \end{pmatrix}$$

**Solution**

$$\begin{aligned} \begin{pmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \end{pmatrix} &= \frac{1}{4} \begin{pmatrix} \omega^0 & \omega^0 & \omega^0 & \omega^0 \\ \omega^0 & \omega^{-1} & \omega^{-2} & \omega^{-3} \\ \omega^0 & \omega^{-2} & \omega^{-4} & \omega^{-6} \\ \omega^0 & \omega^{-3} & \omega^{-6} & \omega^{-9} \end{pmatrix} \mathbf{f} \\ &= \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 4 \\ 9 \end{pmatrix} \\ &= \frac{1}{4} \begin{pmatrix} 14 \\ -4 + 8i \\ -6 \\ -4 - 8i \end{pmatrix} \end{aligned}$$